Problems and Answers in Wave Optics

Downloaded From: http://ebooks.spiedigitallibrary.org/ on 01/30/2013 Terms of Use: http://spiedl.org/terms

Problems and Answers in Wave Optics

Vladimir Ryabukho

SPIE PRESS

Bellingham, Washington USA

Downloaded From: http://ebooks.spiedigitallibrary.org/ on 01/30/2013 Terms of Use: http://spiedl.org/terms

Library of Congress Cataloging-in-Publication Data

Ryabukho, Vladimir P.
Problems and answers in wave optics / Vladimir P. Ryabukho.
p. cm.
Includes bibliographical references and index.
ISBN 978-0-8194-8971-5 (alk. paper)
1. Wave theory of light. 2. Optics--Mathematics. I. Title.
QC403.R93 2011
535'.2--dc23

2011041769

Published by

SPIE
P.O. Box 10
Bellingham, Washington 98227-0010 USA
Phone: +1 360.676.3290
Fax: +1 360.647.1445
Email: Books@spie.org
Web: http://spie.org

Copyright © 2011 Society of Photo-Optical Instrumentation Engineers (SPIE)

All rights reserved. No part of this publication may be reproduced or distributed in any form or by any means without written permission of the publisher.

The content of this book reflects the work and thoughts of the author(s). Every effort has been made to publish reliable and accurate information herein, but the publisher is not responsible for the validity of the information or for any outcomes resulting from reliance thereon.

Printed in the United States of America.
First printing

Downloaded From: http://ebooks.spiedigitallibrary.org/ on 01/30/2013 Terms of Use: http://spiedl.org/terms

Preface

This collection presents problems related to a wide scope of wave phenomena in optics, studied within the framework of the university course of general physics. The problems cover the basic topics of wave optics, i.e., electromagnetic waves and oscillations of optical range, spectral properties of electromagnetic waves, polarization of oscillations and waves, reflection and refraction of light, total internal reflection, optics of anisotropic media and crystal optics, interference of monochromatic and mutually coherent waves, temporal and spatial light coherence, interference of partially coherent light, multiple-beam interference, diffraction of light, diffraction gratings, diffraction of light by volume gratings and acoustic waves, and optics of moving media.

This book is written as a supplement for students studying physics, mathematics, and engineering, including medical physics and engineering, biomedical optics, and biophotonics. The problems are chosen such that their solution supports the study and understanding of the basic concepts of wave optics. This is why the collection contains a sufficient number of relatively simple problems for each topic. Paired with any popular optics textbook, these problems illustrate the principles learned from lectures and lab work. The content of some problems allows them to be used for self-training. Many problems are accompanied by schematic illustrations for clarification, since the study of optical problems is largely associated with visual–spatial perception.

What follows is the result of more than 20 years of experience teaching optics to students at Saratov State University. I greatly appreciate the cooperation, contribution, and support of all of

my students, postgraduate students, and colleagues from the university's Department of Optics and Biophotonics, especially from the head of the department, Professor Valery Tuchin. I am indebted to my colleague Vladimir Derbov, a professor in the Department of Theoretical Physics at Saratov State University, for his English-language assistance. I appreciate his fast and high-quality translation, and his spirit of fruitful collaboration. I would also like to express my gratitude to my wife, Anna, and my son, Peter, for their indispensable support, understanding, and patience during the writing of this book.

Vladimir Ryabukho
October 2011

Contents

Chapter 1 Electromagnetic Waves in the Optical Range

1.1 Equations and parameters of electromagnetic waves

1.1 Estimate the phase velocity v of light propagation in a medium with refractive index $n = 1.5$. Determine the wavelength λ of the light in this medium if the frequency of the light oscillations is $\nu = 5 \cdot 10^{14}$ Hz.

1.2 Determine the change Δv of the phase velocity of light propagation due to refraction at the interface between glass ($n_1 = 1.5$) and water ($n_2 = 1.33$), and the ratio of wavelengths λ_1/λ_2.

1.3 Determine the phase velocity v of light propagation in a medium with relative permittivity $\varepsilon = 2.5$ and permeability $\mu = 1$.

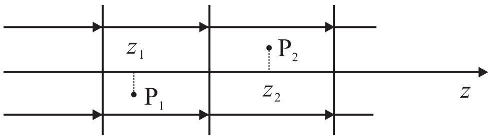
1.4 Show that the plane-wave expression $E(z,t) = E_0 \cos(\omega t - kz)$ is a solution of the wave equation and express the phase velocity of the wave v in terms of the wave parameters, namely, circular frequency ω and spatial circular frequency (wave number) k.

1.5 Using the complex form of the expression for a plane harmonic wave propagating along the z axis, show that this expression satisfies the wave equation.

1.6 Using the complex expression for the plane harmonic electromagnetic wave and Maxwell's equations in the differential form, show that the vectors $\vec{E}$, $\vec{H}$ and $\vec{k}$ of the wave form the right-hand trio of mutually perpendicular vectors in isotropic dielectric media.

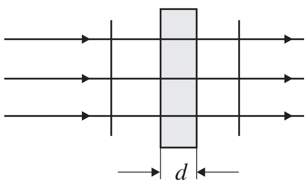
1.7 The plane-harmonic wave in a certain medium is described by the expression $E(z,t) = 5\cos\left(2\pi \cdot 5 \cdot 10^{14} \cdot t - \frac{2\pi}{0.4 \cdot 10^{-6}} \cdot z\right)$. Determine the circular frequency ω, the wavelength λ, the phase velocity of the wave v, and the refractive index of the medium n.

1.8 Determine the phase difference $\Delta\phi_{12}$ between the oscillations excited by the plane wave with wavelength λ, propagating along the z axis, at points P_1 and P_2 with coordinates z_1 and z_2.

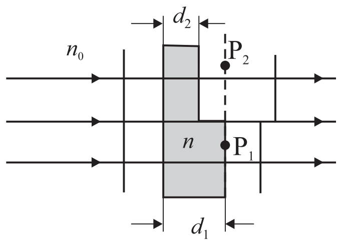


1.9 Determine the phase shift $\Delta\phi$ of the wave with frequency ν in the course of its propagation at the distance Δz in the medium with refractive index n.

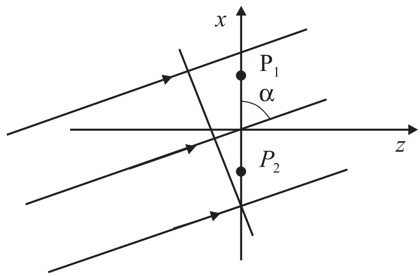
1.10 A plane wave having the wavelength $\lambda_0 = 600$ nm in vacuum is normally incident on the plane-parallel plate with thicknesses $d = 2$ mm and refractive index $n = 1.75$. Determine the phase difference $\Delta\phi$ between the oscillations at the front and back surfaces of the plate.



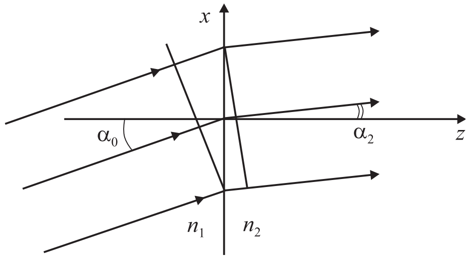
1.11 A plane wave with free-space wavelength λ_0 is normally incident on a plane-parallel glass plate of stepped thickness d_1 and d_2 and refractive index n. The plate is submerged in a liquid with refractive index n_0. Determine the phase difference $\Delta\phi_{12}$ between the wave oscillations at points P_1 and P_2, lying in the plane of the back surface of the plate with greater thickness d_1.



1.12 A plane wave with wavelength λ is incident on the plane (x, y) perpendicular to the y axis and forming angle α with the x axis. Write the expression for the spatial distribution of the wave phase $\phi(x)$ along the x axis if, at the origin of coordinates, the phase of the wave $\phi(x = 0) = \pi/2$. Determine the phase difference $\Delta\phi_{12}$ between the oscillations at the points P_1 and P_2 of the x axis if the difference of their coordinates is Δx_{12}.

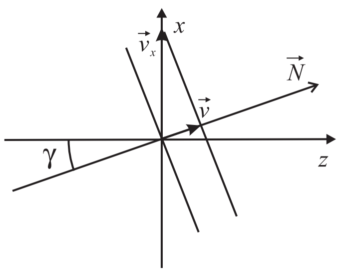


1.13 A plane wave with wavelength λ_0 is incident with incidence angle α_0 onto the interface of two dielectrics with refraction indices n_1 and n_2. Derive the expression for the spatial phase distribution for the incident $\phi_1(x)$ and refracted $\phi_2(x)$ waves versus the x coordinate at the boundary between the media if the incidence plane coincides with the (x, z) plane.

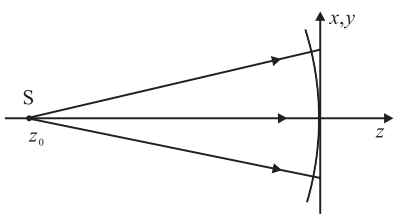


1.14 Determine the phase difference $\Delta\phi_{12}$ of the oscillations excited by the monochromatic plane wave with wavelength λ at points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ in the (x, y) plane, at which the wave is incident, forming the angles α and β with axes x and y.

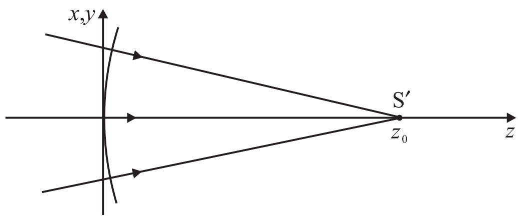
1.15 A monochromatic plane wave propagates in the plane (x, z) in a direction forming the angle $\gamma = 30\,\text{deg}$ with the z axis. Determine the phase velocity v_x of the wave in the x-axis direction, provided that the refractive index of the medium $n = 1$.



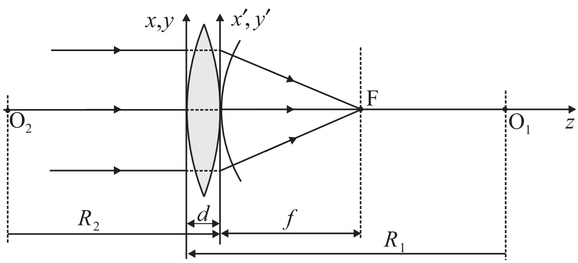
1.16 A spherical wave from the point source S, located at the point z_0 and belonging to the z axis, is incident on the (x, y) plane. Using the paraxial (parabolic) approximation, write the expression for the spatial distribution of the wave phase $\phi(x, y)$ in the (x, y) plane if the initial phase of the wave at point z_0 is $\phi(0, 0, z_0) = \pi$.



1.17 A convergent spherical wave is incident on the (x, y) plane and focused at point S' of the z axis with coordinates $(0, 0, z_0)$. Using the paraxial (parabolic) approximation, write the equation describing the spatial distribution of wave phase $\phi(x, y)$ in the (x, y) plane if, at the origin of the coordinate frame in the (x, y) plane, $\phi(0, 0) = \pi$.

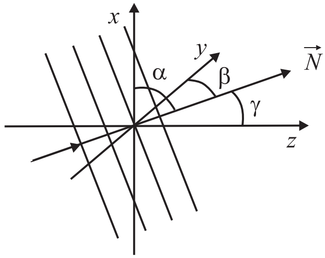


1.18 A plane wave is incident on the focusing lens parallel to its optical axis. Assuming the lens to be thin [lens thickness d is small; the curvature radii of the lens spherical surfaces R_1 and R_2 are large; and the coordinates (x, y) of the ray incident on the lens are approximately equal to coordinates (x', y') of the ray emerging from the lens], and using the paraxial approximation, derive the equation describing the spatial distribution of the phase $\phi(x', y')$ of the wave in the plane (x', y') located immediately behind the lens, and express focal length f of the thin lens in terms of R_1, R_2, and the refractive index n of the lens surrounded by air with refractive index $n_0 = 1$.



1.19 A plane wave with wavelength λ propagates in the direction $\vec{N}(\cos\alpha, \cos\beta, \cos\gamma)$, forming the angles α, β, γ with the axes of the rectangular coordinate system x, y, z, respectively.

Determine the spatial periods $\Lambda_x, \Lambda_y, \Lambda_z$ and the spatial frequencies f_x, f_y, f_z of the wave along axes x, y, z, respectively.



1.20 Determine spatial frequency f_z along the z axis for the plane wave with circular temporal frequency ω, forming angle γ with the direction of wave propagation.

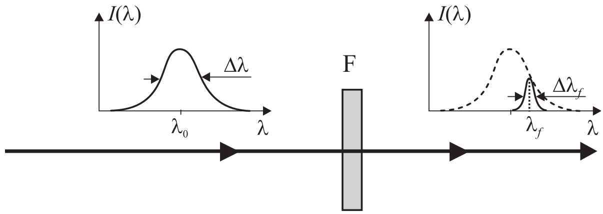
1.2 Spectral properties of electromagnetic waves

1.21 The spectral contour of quasi-monochromatic light has a half-width $\Delta\lambda \approx 0.01$ μm at the central wavelength $\lambda_0 \approx$ 600 nm. Determine the temporal coherence length l_c and the coherence time τ_c of such light.

1.22 Determine the coherence time τ_c and the length of the wave train l_c of the electromagnetic wave in a medium with refractive index $n = 1.5$ if the half-width of the frequency contour of this wave is $\Delta\nu \approx 10^{13}$ Hz.

1.23 Determine the mean number of oscillations m in a single-wave train of the radiation of a red light-emitting diode with mean wavelength $\lambda_0 \approx 0.65$ μm and spectral contour width $\Delta\lambda \approx 20$ nm, and the radiation of a helium-neon gas laser with $\lambda_0 \approx 0.63$ μm and $\Delta\lambda \approx 0.04$ nm.

1.24 White light is passed through an optical filter F. The spectral-contour width of the white light is $\Delta\lambda \approx 150$ nm and the central wavelength is $\lambda_0 \approx 0.55$ μm. The central wavelength of the filter transmission band is $\lambda_f \approx 0.65$ μm and transmission bandwidth is $\Delta\lambda_f \approx 15$ nm. Determine the ratio of coherence lengths (lengths of wave train) after and before the filter.



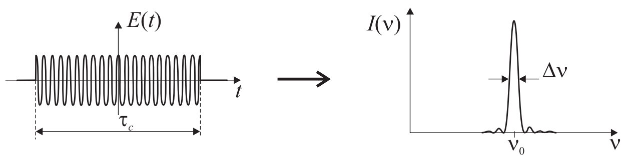
1.25 The temporal coherence length of a quasi-monochromatic light is $l_c \approx 30\ \mu\text{m}$ with mean wavelength $\lambda_0 \approx 600$ nm. Determine the width of the spectral contour in the wavelength scale $\Delta\lambda$ and in the frequency scale $\Delta\nu$ for this light.

1.26 The spectral contour of a quasi-monochromatic light is described by the Gaussian function $I(\lambda) = I_0 \exp[-(\lambda - 0.6)^2 \times 10^4]$, where the wavelength λ is expressed in micrometers. Determine the temporal coherence length l_c for this light.

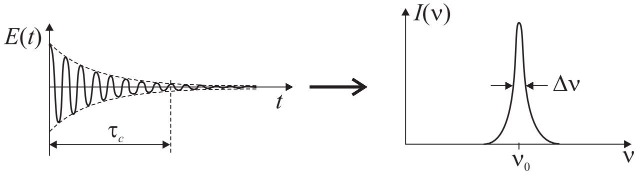
1.27 Derive an expression for the intensity of the frequency spectrum $I(\nu) = |E(\nu)|^2$ of the wave train of harmonic oscillations having the finite duration τ_c

$$E(t) = \begin{cases} E_0 \cos(2\pi\nu_0 t + \phi_0), & -\tau_c/2 \le t \le \tau_c/2, \\ 0, \quad |t| > \tau_c/2. \end{cases}$$

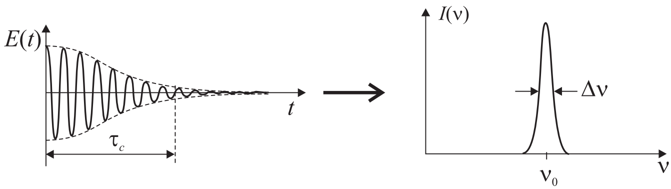
Determine the spectral-contour width $\Delta\nu$ at the half-maximum depending on τ_c.



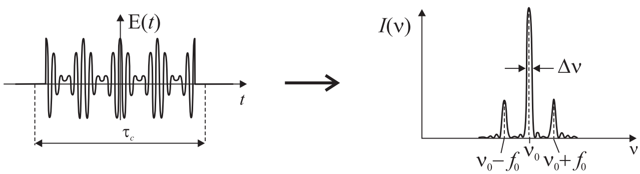
1.28 Derive the expression for the intensity of the frequency spectrum $I(\nu) = |E(\nu)|^2$ of the wave train of damped oscillations: $E(t) = E_0 \exp(-t/\tau_c) \cos(2\pi\nu_0 t)$, where τ_c is the damping time. Determine the width of the spectral contour $\Delta\nu$ at the half-maximum depending on τ_c.



1.29 Derive the expression for the intensity of frequency spectrum $I(\nu) = |E(\nu)|^2$ of the wave train of oscillations, damped according to the Gaussian law, i.e., $E(t) = E_0 \exp[-(t/\tau_c)^2] \cos(2\pi\nu_0 t)$, where τ_c is the damping time. Determine the spectral contour width $\Delta\nu$ at half-maximum depending on τ_c.



1.30 Derive the expression for the intensity of frequency spectrum $I(\nu) = |E(\nu)|^2$ of the finite-length wave train of oscillations, modulated according to the harmonic law $E(t) = E_0 0.5[1 + m \cos(2\pi f_0 t)] \cos(2\pi\nu_0 t)$, where m is the modulation coefficient, f_0 is the modulation frequency, and $f_0 \gg 1/\tau_c$.



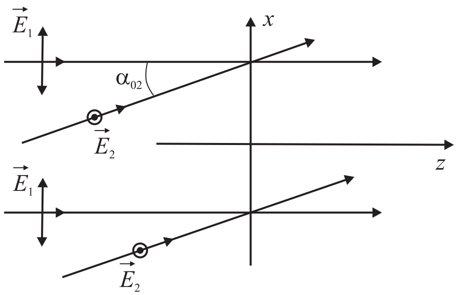
1.3 Polarization of electromagnetic waves

1.31 Show that the superposition of two linearly polarized plane waves with orthogonal directions of field oscillation $E_x(z,t)$ and $E_y(z,t)$, having the same frequency and arbitrary phase difference $\Delta\phi_{xy}$, yields an elliptically polarized wave.

1.32 Show that a linearly polarized plane wave $\vec{E}(t,z)$ may be presented as a superposition of two circularly polarized plane waves $\vec{E}_L(t, z)$ and $\vec{E}_R(t, z)$ with opposite directions of rotation of the electric field vector.

1.33 What are the phase difference $\Delta\phi_{xy}$ and the amplitudes E_{ox} and E_{oy} of two orthogonally polarized waves that provide a resulting wave with (a) linear and (b) circular polarization?

1.34 Two monochromatic plane waves with wavelengths λ = 600 nm linearly polarized in orthogonal directions are incident on the (x, y) plane under different angles with the x axis, $\alpha_{01} = 0$ deg and $\alpha_{02} = 30$ deg. Determine the spatial period Λ along the x axis for the variation of the polarization state of resulting summary oscillations excited by these waves.



1.35 Determine the polarization state of the electromagnetic oscillations arising as a result of superposition of two linearly polarized monochromatic oscillations $E_x(z, t) = E_{0x}\cos(\omega_{0x}t + \phi_{0x})$ and $E_y(z, t) = E_{0y}\cos(\omega_{0y}t + \phi_{0y})$ with orthogonal directions of oscillations and different frequencies ω_{0x} and ω_{0y}.

1.36 Determine the polarization state of the plane electromagnetic wave that appears as a result of superposition of two linearly polarized plane waves $E_x(z, t) = E_{0x}\cos[\omega_0 t - kz + \phi_{0x}(t)]$ and $E_y(z, t) = E_{0y}\cos[\omega_0 t - kz + \phi_{0y}(t)]$ with orthogonal directions of oscillations and the initial phases $\phi_{0x}(t)$ and $\phi_{0y}(t)$ randomly varying with time.

1.4 Energy, power, and intensity of light

1.37 The electric field strength E of a plane electromagnetic wave varies following the expression $E(z, t) = 20\cos(2\pi\cdot 4\cdot 10^{14}\cdot t-$

$\frac{2\pi}{5\cdot10^{-7}} \cdot z + \pi/2)$ V/m. Determine the frequency ν, wavelength λ, and velocity v of the wave. Write an explicit expression for the magnitude of the Pointing vector $\vec{S}$ and calculate the value of the intensity I [W/m^2] of this wave in a medium with refractive index $n = 1.5$.

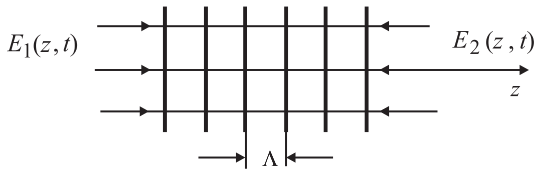
1.38 The amplitude of the electric field strength in a laser beam of light in a medium with refractive index $n = 1.5$ equals $E_0 = 400$ V/m. Determine the intensity of this beam of light I [W/m^2] and its power P [mW], assuming this beam to be collimated with the cross section diameter $D = 5$ mm and to have a uniform distribution of intensity in the cross section of the beam.

1.39 The power of a laser beam of light is $P = 2$ mW. The distribution of the light intensity over a certain cross section of the beam is given by the Gaussian function $I(x,y) = I_0 \exp[-(x^2 + y^2)/w^2]$ [W/m^2]. Determine the intensity I_0 [W/m^2] and the amplitude E_0 [V/m] of the electric field strength in the beam center if the beam radius $w = 1$ mm and the refractive index of the medium $n = 1.33$.

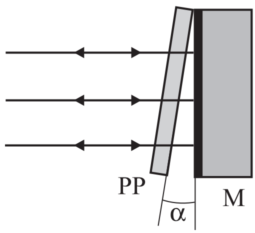
1.40 The distribution of the electric field strength $E(x,y)$ over the cross section x, y of a laser beam in air ($n = 1$) is described by the Gaussian function $E(x,y) = E_0 \exp[-(x^2+y^2)/w_0^2]$. Write the expression for the distribution of field intensity $I(x,y)$ over the cross section of the laser beam and estimate the beam power P [mW] if $E_0 = 500$ V/m and the beam radius with respect to the field amplitude distribution $w_0 = 3$ mm.

1.5 Standing electromagnetic waves

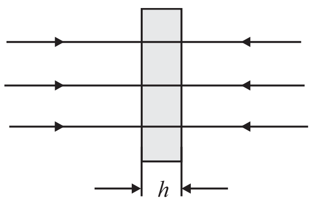
1.41 Determine the separation Λ between the adjacent antinodes and nodes of a standing wave, formed in a medium with refractive index $n = 1.33$ under the superposition of counter-propagating waves $E_1(z,t)$ and $E_2(z,t)$ with the free-space wavelength $\lambda_0 = 0.6$ µm.



1.42 Determine the spatial period Λ_x of photographic-plate-blackening fringes in the Wiener experiment on photographic detection of standing electromagnetic waves; the light used in the experiment has wavelength $\lambda_0 = 560$ nm, and the photosensitive layer PP forms the angle $\alpha = 1$ deg with the plane of the metallic mirror M.



1.43 Determine the number of antinodes m of a standing wave and, correspondingly, the number of blackening layers formed in a negative photosensitive layer with refractive index $n = 1.35$ and thickness $h = 8$ μm placed perpendicularly to the directions of propagation of two counter-propagating plane-monochromatic waves with frequency $5 \cdot 10^8$ MHz.



1.44 A photosensitive layer with refractive index $n = 1.5$ is coating a metallic mirror M, on which a collimated laser beam with the wavelength $\lambda_0 = 633$ nm is incident on the angle $\alpha_0 = 45$ deg. Determine the separation Λ between the antinodes of the standing wave produced by the superposi-