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DYNAMICS OF SATELLITES

EDITOR

MAURICE ROY

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DYNAMIQUE DES SATELLITES

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Préface

Depuis le lancement de SPOUTNIK I par l'Union Soviétique le 4 Octobre 1957, des expériences humaines de Mécanique céleste de cette sorte ont été répétées à de nombreuses reprises en U.R.S.S. et aux U.S.A.

En 1961, sur ma proposition, l'Union Internationale de Mécanique théorique et appliquée retint l'idée de consacrer en 1962 un Symposium spécial à la confrontation des résultats des expériences soviétiques et américaines en vue d'en tirer le maximum d'enseignements sur la question fondamentale suivante concernant la « Dynamique des satellites artificiels » de la Terre: quelles sont la nature et les lois des forces réelles qui agissent sur ces mobiles au voisinage de notre planète, et qui déterminent par conséquent leur mouvement?

En d'autres termes, il s'agissait de faire le point de nos connaissances sur le problème du mouvement des Astres, magistralement résolu par NEWTON il y a plus de trois siècles pour des astres quasi-ponctuels et assez éloignés. Les moyens d'observation utilisés pour connaître avec la meilleure précision possible le mouvement des satellites artificiels lancés depuis 1957, et le fait de la proximité relative de ces satellites par rapport à la Terre sont par eux-mêmes de nature à révéler soit des altérations de la loi classique de l'attraction newtonienne, dont la signification serait à rechercher, soit l'intervention de forces perturbatrices, dont l'origine et l'expression seraient à préciser.

Ce sont l'examen et la discussion de ces problèmes qui constituèrent l'objet du Symposium en question, dont le Comité Scientifique fut composé du Dr. H. L. DRYDEN, du Dr. D. G. KING-HELE, du Pr. G. C. McVITTIE, du Pr. LÉONID I. SEDOV, et du signataire de ces lignes, à qui en fut confiée, avec l'honneur de la présidence, la responsabilité de l'organisation à Paris.

Par sa conception même, ce Symposium postulait l'accord des savants soviétiques et américains, dont la participation était essentielle. C'est un heureux privilège de I.U.T.A.M. que d'avoir pu obtenir sans difficulté cet accord, et je tiens ici à exprimer au Dr. H. DRYDEN et au Pr. L. SEDOV, l'un et l'autre si connus des savants de la Mécanique en tous les pays, pour la contribution si active et si efficace qu'ils ont apportée à notre Comité organisateur.

Comme tous les Symposiums I.U.T.A.M., celui-ci était réservé à un nombre restreint de participants, auteurs ou observateurs, tous invités par le Comité. En fait, s'agissant d'un problème fondamental mais étroitement limité, il fut décidé de borner en définitive à 38 le nombre des participants et à 25 le nombre des communications, dont 1 d'origine A. (All.), 3 d'origine U.K., 10 d'origine U.R.S.S. et 11 d'origine U.S.A.

Les séances de travail se sont tenues les 28, 29 et 30 Mai 1962, à Paris, dans des locaux réservés et spécialement aménagés de la Maison de la Chimie, offrant aux participants un cadre d'intimité de travail favorable à des échanges de vues animés.

Sans entrer dans le détail des études que le Lecteur trouvera plus loin, bornons-nous à mentionner ici quelques conclusions.

La loi de NEWTON de l'attraction universelle reste inébranlée, étant retenue comme l'expression la plus sûre de la force de gravitation pour prévoir la trajectoire des satellites artificiels, et pour rattacher à d'autres causes les altérations constatées de leurs trajectoires.

L'effet de relativité générale a été traité par un auteur et reconnu négligeable.

Le couplage entre les mouvements du centre d'inertie et autour de celui-ci a été discuté et son très minime effet réel a été apprécié, tandis qu'une intervention non négligeable d'un couple d'origine magnétique dans le mouvement autour du centre d'inertie a été reconnu possible, concurremment avec celle du très faible couple d'origine gravitationnelle.

Les forces extérieures de surface, ou « résistances » telles que la résistance de milieu ou celle de charge électrique, ont été discutées et leurs appréciations comparées. Ces forces sont extrêmement minimes, mais l'effet plus ou moins séculaire des perturbations qu'elles occasionnent est notable, surtout pour les satellites de basse altitude.

L'interprétation des résultats accumulés depuis quatre ans et demi, et déjà excessivement nombreux, entraîne une profonde révision des supputations antérieures sur la nature, la composition et les propriétés, notamment de densité et de température, de la très haute atmosphère et de l'espace environnant.

Le Symposium de Paris marquera sans doute une étape dans le progrès des connaissances scientifiques relatives à ces problèmes fondamentaux de la Dynamique des Satellites artificiels, problèmes dont l'importance est primordiale pour la future exploration humaine de l'Espace extra-atmosphérique.

Le présent volume, dont la Maison Springer a assuré l'édition avec le soin qui a fondé sa réputation et avec une diligence dont je tiens à la remercier ici, contient la totalité des communications présentées au Symposium. La délégation russe a bien voulu fournir des traductions, en anglais ou en français, de ses contributions et je l'en remercie hautement.

Pour sa part, l'éditeur aurait souhaité pouvoir publier toutes les discussions, si intéressantes, qui ont marqué les séances de travail. Mais, ceux qui connaissent par expérience les inévitables difficultés que l'on rencontre pour coordonner, avec le concours des intéressés et en vue de leur publication sans retard, les différentes parties de telles discussions comprendront certainement pourquoi, notamment pour hâter l'édition, je n'ai pu réaliser le projet envisagé. Cependant, à la demande des intéressés, quelques interventions ont pu être reproduites dans ce volume, à la suite des communications concernées.

La plupart des participants, dont la liste alphabétique est la suivante:

MM. B. ANDERSON, R. BAKER, Y. BATRAKOV, L. BROER, D. BROUWER, P. CONTENSOU, M. DAVIES, G. DOUBOCHINE, H. DRYDEN, B. GARFINKEL, L. GAUTHIER, S. HERRICK, W. IRVINE, L. JACCHIA, W. KAULA, D. KING-HELE, Y. KITAEV, D. KULIKOV, J. KOVALEVSKY, Y. KOZAI, K. MAGNUS, B. MAY, D. MCDERMOTT, G. MCVITTIE, R. MERSON, A. MOLTCHANOV, P. MUSEN, R. NAUMANN, W. PRIESTER, L. PUYEO, H. ROBE, M. ROY, W. SCHULZ, L. SEDOV, I. SHAPIRO, G. TEMPLE, S. WYATT,

ont prise une part trés active aux échanges de vues et discussions, et une trace invisible en subsiste dans les retouches apportées, en définitive, par plusieurs auteurs à leurs textes primitifs.

Ce sont à tous ces participants, auteurs de rapports ou intervenants des discussions, que j'exprime à nouveau la gratitude du Comité pour leur active coopération, pour la valeur scientifique de leur concours, enfin pour leur esprit de collaboration cordiale et directe. En particulier, je remercie à nouveau le Dr. KOVALEVSKY et le Dr. MUSEN qui ont bien voulu opérer aimablement, lorsqu'il apparaissait utile, la conversion de la langue russe en anglais ou en français, et réciproquement.

Le Symposium a été honoré de la présence du Pr. G. TEMPLE, Président de I.U.T.A.M., et du Pr. L. GAUTHIER, Vice-Doyen de la Faculté des Sciences de Paris. Les Dames ont pu tirer quelque plaisir de visites organisées à leur intention pour occuper chaque jour une partie de leur aprés-midi. Un dîner intime mais cordial, organisé le 29 Mai au Bois de Boulogne grâce à une aide très obligeante de la Direction des Relations Culturelles du Ministère français des Affaires Etrangères, m'a permis d'exprimer à nos hôtes l'espoir qu'ils garderaient bon et utile souvenir de leur séjour studieux à Paris.

Je voudrais encore, en terminant cette introduction, citer les organisation industrielles françaises qui, en outre d'une aide particulière accordée par le C.N.R.S., ont bien voulu m'apporter une assistance bénévole, généreuse, et d'une très haute utilité pour notre organisation matérielle: Société française Babcock et Wilcox, Chambre Syndicale des Producteurs d'Aciers Fins et Spéciaux, Compagnie de Péchiney, Cie Générale de Télégraphie Sans Fil, Société d'Electro-Métallurgie d'Ugine, et Cie Fse Thomson-Houston.

Paris, Mars 1963

Maurice Roy

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The elimination of spurious data in the process of preliminary and definitive orbit determination

By

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Abstract. Since the advent of high-precision electronic and electro-optical instruments, considerable research has been carried out in new methods of preliminary orbit determination. Early procedures for processing mini-track, pulsed-radar, and BAKER-NUNN camera data are briefly summarized. A more novel method, employing high-accuracy, range-rate data together with low-accuracy, angle data is also presented and its application to a recent Ranger lunar probe orbit, as observed by GOLDSTONE, is noted.

The process of eliminating spurious observational data on the basis of these preliminary orbits and through the use of a filtering technique is considered in detail. The potential advantages of the Gibbsian method for correlating observed data are also noted.

I. Historical background

An exciting requirement for orbit determination arose in 1801 when GAUSS was challenged by the problem of "rediscovering" the minor planet Ceres. The track of this first minor planet had been lost after a few initial observations. Up to this time, there had existed no great fear of "losing" a major planet, and the demands for rigorous methods of orbit determination from terrestrial observations were not really apparent.

As might be expected, even NEWTON had concerned himself with the problem of orbit determination and had devised graphical solutions; while EULER in 1744 published the first complete non-graphical solution. OLBERS' method for determining parabolic orbits (1797) has stood the test of time and still finds general application. In spite of these earlier approaches, including even the initial efforts of LAMBERT, LAGRANGE, and LAPLACE, we owe to GAUSS the most exhaustive and useful early research into the field of orbit determination. More recently GIBBS, LEUSCHNER, and HERRICK have provided us with significant advances in precision numerical techniques for definitive orbit determination and improvement.

The classical problem facing astronomers of the past was rather simple to formulate: given a limited number of tediously reduced angular measurements of an object from one or more observation stations on the Earth made at specific times, what were the elements of the orbit upon which the object was travelling? Provided with these orbital elements, the astronomer could generate an ephemeris accurately by the method of general or special perturbations or approximately by utilizing two-body formulae.

However, complete position fixes of the object werenever available to the astronomer; i. e., no range or range-rate observations were available. These data had somehow to be derived or estimated in the orbit computation technique itself. Such a situation may well be the reverse of the one confronting modern astrodynamics, in which a vast quantity of relatively easily reduced range and range-rate data are often accurately known, whereas angular position data may be unknown or known only to an inferior degree of precision.

Classical orbit determination procedures can generally be categorized as Gaussian, Gibbsian, Lagrangian, or Laplacian. For the purposes of this paper, only the method due to LAPLACE is dealt with in detail, and the interested reader's attention is directed to Ref. [*1*, *2*] and [*3*] for a more complete exposition of this and the other methods.

The application of classical orbit-determination schemes such as the Laplacian to the preliminary determination of orbits from range and/or range-rate data is not particularly obvious. Many special-purpose schemes have been devised by scientists faced with the immediate problem of processing DOPPLER data. Some of these procedures make use of the remarkable inflection point that is found in a plot of range rate versus time. Typical of this method are the versions discussed by V. A. KOTELNIKOV [*4*] and C. S. LORENS [*5*]. The time of the inflection point (i. e., when $\dot{\varrho} = 0$) coincides with the moment when the distance between the satellite and the observation station is minimum. This time can be graphically estimated to perhaps 0.2 second and to a somewhat higher accuracy numerically; e. g., on p. 261 of reference 5 a quartic interpolation formula is utilized to find the time and distance of closest approach at the inflection point. Given the slope $\ddot{\varrho}$ of the $\dot{\varrho}$-versus-time plot at this time and the assumption of either a horizontally rectilinear or a circular orbit, the speed of the vehicle and its minimum distance from the DOPPLER station can be estimated.

W. A. KEMPER [*6*] has utilized a least-squares fit to obtain a position fix of an object by triangulation, using simultaneous (i. e., synoptic) radar data obtained from two different stations. A smoothing process based upon a number of these complete "fixes" is employed to obtain satellite position, velocity, and acceleration.

I. HARRIS and R. JASTROW [7] establish angular measures of a satellite passing by a mini-track station with antennae placed along E–W and N–S baselines. From the phase difference of incoming radio waves from a satellite measured between these antennae, the angle to the satellite, α, is obtained to an accuracy of H/r_c (where H is the height of the satellite, and r_c is the geocentric distance of the mini-track station). An iterative process is used to solve for α, H, and $\dot{s}$; substitution of these quantities into the *vis-viva* integral and into the equation of a conic yields the semi-major axis a, the eccentricity e, and the argument of perifocus ω.

Other methods, somewhat similar to the foregoing, are described in Ref. [6] and [8]. Useful as these methods may be for carrying out the tasks for which they were designed, they are only stop-gap, special-purpose procedures and should be replaced by more carefully devised, general-purpose schemes that take optimum advantage of the carefully contrived methods devised by LAPLACE, LAGRANGE, GAUSS, GIBBS, and others.

In more specific terms most present systems for orbit determination (see Ref. [9–17]) suffer from one or more of the following difficulties:

1. Accurate range or range-rate data are degraded or contaminated by inferior angular data (e. g., ± 40 feet in range error for a space vehicle at a height of one Earth's radius is roughly equivalent to plus or minus one microradian or 0.2 second arc error in angle; moreover, the ratio increases with height).

2. Synoptic data are required when observations are made from two or more stations in order to "simplify" or render possible the orbit determination.

3. Dynamical relationships are ignored entirely when the orbit is determined from geometrical fixes obtained, e. g., by the intersection of three surfaces of revolution in space.

4. If two or more uncooperative or passive satellites (or empty boosters or carriers, etc.) make passes simultaneously, then there exists the possibility of ambiguity in the correlation of the data collected.

5. Some systems, although proved to be satisfactory for the determination of low-altitude satellites, are not necessarily applicable to high-altitude satellites and space probes. As C. S. LORENS [5] states, "The use of (D)oppler tracking is limited principally to satellites or projectiles which come close to the Earth."

6. An uninterrupted sequence of data over a complete pass, i. e., data sufficient to define the inflection point, is required.

In addition to the foregoing deficiencies, many researchers fail to distinguish clearly between orbit determination and orbit prediction.

The initial conditions for orbit prediction are provided by orbit determination. Thus the question of the realism with which a certain procedure can describe the future orbit of a space vehicle, i. e., produce an ephemeris, is a question apart from that of the initial orbit determination. Certainly orbit determination and orbit prediction go hand in hand to produce an ephemeris; but the capability and accuracy of each should be assessed individually.

An analogous problem to that of the confusion between orbit determination and prediction is that in many orbit-determination schemes the differences between the preliminary orbit and subsequent, improved orbits are not clearly comprehended. All too frequently many details are incorrectly included in the generation of the preliminary orbit. It should be kept in mind that the preliminary orbit is indeed preliminary, and that it would often be inconsistent to take into account such details as the correction for the index of refraction and other deleterious ionospheric effects[1], the complex smoothing or normal-place procedures for processing data, or the inclusion of the secular perturbation in certain orbital elements in the preliminary orbit. The proper place for most of these refinements is in the differential correction procedure.

More recently DEUTSCH [*18*] has considered the use of the DOPPLER method for determining the orbits of artificial satellites. He calls upon the spectroscopic, binary-star technique and utilizes it in a new sense for the determination of a planetary satellite carrying a frequency-stabilized transmitter or transponder. DEUTSCH can obtain orbital elements (including the planetary mass) to $\pm$ 1 per cent. PAUL KOSKELA [*19*] and BRUCE DOUGLAS [*20*] are independently extending this work into elliptical orbits with perturbations. Also studying this new area of electronic data are HARRIS and CAHILL [*21*] who used a GAUSS-OLBERS procedure based on three or more complete fixes in two to four minutes of observation time.

PATTON [*22*] also considers orbit determination from a single-pass DOPPLER observation, but instead of remaining with the preliminary orbit problem, he goes into a standard least-squares fit in a differential correction technique. He assumes that a preliminary orbit has somehow already been conceived. One of his comments is particularly noteworthy: "It has been found that parameters consisting of position and velocity components for a given time readily yield a convergent solution . . ." The proper choice of parameters assumes considerable importance in the differential correction procedure subsequent to the preliminary orbit generation.

[1] In this same regard, it is noted that ionospheric refraction usually causes greater inaccuracies in angular than in range or range-rate data — another reason for not utilizing these data together.

II. Classical methods

Laplacian method. Perhaps the most well-known classical method for preliminary orbit determination is the one due to LAPLACE. The method depends upon the acquisition of three sets of α and δ obtained, e. g., by the reduction of BAKER-NUNN camera films. From these, by numerical differentiation, one generates the first and second derivatives of the unit vector $\boldsymbol{L}$ reckoned at the second date. (See Vol. II of HERRICK's Astrodynamics [2], or see pages 131 through 135 of Ref. [1] for a derivation of this method.) The four fundamental equations of the Laplacian method are

$$\left(\frac{\mu \dot{\varrho}}{r^3} + \ddot{\varrho}\right) \boldsymbol{L} + 2 \dot{\varrho} \dot{\boldsymbol{L}} + \varrho \ddot{\boldsymbol{L}} = \ddot{\boldsymbol{R}} + \frac{\mu \boldsymbol{R}}{r^3} \tag{1}$$

(called the POINCARÉ form and having three components), and

$$r^2 = \varrho^2 - 2 \varrho (\boldsymbol{L} \cdot \boldsymbol{R}) + R^2 . \tag{2}$$

(All notation in these equations is for the most part standard and is defined in Ref. [1].)

These four equations include the four unknowns ϱ, $\dot{\varrho}$, $\ddot{\varrho}$, and r. The quantities $\dot{\varrho}$ and $\ddot{\varrho}$ can be eliminated from Eq. (1) to reduce these three equations in four unknowns to one equation in two unknowns:

$$D \varrho = A' - \frac{B'}{r^3} .$$

Eqs. (2) and (3) can then be solved simultaneously, e. g., by NEWTON's method, for ϱ and r at the second date. Three major difficulties are found in the application of this method to space vehicle observations:

1. For nearly great circle paths on the celestial sphere, $\dot{\boldsymbol{L}}$ is perpendicular to $\boldsymbol{L}$ and $\ddot{\boldsymbol{L}}$ so that D goes to zero, and A' and B' are poorly defined. As suggested by S. HERRICK, this difficulty can often be partially circumvented by solving Eq. (3) directly for r under the assumption that D equals zero.

2. Because the numerical differentiation is expanded about the second date, the Laplacian orbit represents observations well at this date, but not at the first and third dates. This difficulty can be circumvented through the use of a LEUSCHNER differential correction (see pages 146–150 of Ref. [1]).

3. If the angular data are noisy, as indicated by erratic and often large second differences, the $\dot{\boldsymbol{L}}$ and especially $\ddot{\boldsymbol{L}}$ are poorly determined. There is no way to avoid this difficulty, and because of it the Laplacian method is ordinarily abandoned in satellite work when α and δ are not known to a high degree of accuracy. Thus, except for high-precision

reduction of BAKER-NUNN camera films or astronomical photographic plates, the method is ordinarily not useful.[1]

Lagrange-Gauss-Gibbs first approximation (after S. Herrick). Prior to the employment of the Lagrangian, Gaussian, or Gibbsian preliminary orbit method, a first-approximation procedure must be employed to find the ϱ's at three dates. A unified first-approximation procedure has been developed by HERRICK that is termed the "LAGRANGE-GAUSS-GIBBS first approximation." The fundamental equations are

$$c_1\,\varrho_1\,\boldsymbol{L} - \varrho_2\,\boldsymbol{L}_2 + c_3\,\varrho_3\,\boldsymbol{L}_3 = c_1\,\boldsymbol{R}_1 - \boldsymbol{R}_2 + c_3\,\boldsymbol{R}_3$$

$$c_i = A_i + \frac{B_i'}{r_2^3} + \cdots, \tag{5}$$

and Eq. (2). (See Ref. [2] for the derivation.)

The $A_i'\,s$ and $B_i'\,s$ are functions of the time intervals between observations. Equations (2), (4), and (5) may be solved for ϱ_1, ϱ_2, ϱ_3, and r_2. Many of the major difficulties encountered in the Laplacian method are absent in the LAGRANGE-GAUSS-GIBBS first approximation. For deep space probes, however, in which $\boldsymbol{L}_1$, $\boldsymbol{L}_2$, and $\boldsymbol{L}_3$ are close together and inaccurately known, the method is sometimes unusable. Furthermore, Eqs. (5) are truncated series expessions which may lead to errors if the observations are spaced far apart in time and if r_2 is not large.

III. Selected new methods

As an illustrative example of the special demands occassioned by satellite probe orbits, let us consider the definition of an orbit given relatively inaccurate and only slightly changing values of α and δ and a relatively accurate value of $\dot{\varrho}$. This is precisely the type of information available from many of the United States deep space tracking stations such as GOLDSTONE (operated by the Jet Propulsion Laboratory of NASA) and e. g., obtained from the Ranger lunar probe tracking.

Modified Lapacian method [23]. If one eliminates $\dot{\varrho}$ rather than ϱ and $\ddot{\varrho}$ from Eq. (1), by taking the dot product of Eq. (1) with L^2,

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[1] JEAN KOVALESKI commented that angular data from more than the minimum three dates have been used effectively by him to circumvent the use of intermediate elements. He represents all of the angular data (as obtained from photographic plates) by a polynomial i. e.,

$$\boldsymbol{L}_i = \boldsymbol{L}_2 + \dot{\boldsymbol{L}}_2\,\tau_i + \ddot{\boldsymbol{L}}_2\,\tau_i^2/2! + \dddot{\boldsymbol{L}}_2\,\tau_i^3/3! + \cdots,$$

and obtains very accurate $\dot{\boldsymbol{L}}$ for the Laplacian method from which point he proceeds directly to the differential correction [30].

[2] Note that $\boldsymbol{L}\cdot\dot{\boldsymbol{L}}$ always remains equal to zero, $\dfrac{d(\boldsymbol{L}\cdot\dot{\boldsymbol{L}})}{d\tau} = \dot{\boldsymbol{L}}\cdot\dot{\boldsymbol{L}} + \boldsymbol{L}\cdot\ddot{\boldsymbol{L}} = 0$ or $\boldsymbol{L}\cdot\ddot{\boldsymbol{L}} \equiv -\dot{\boldsymbol{L}}\cdot\dot{\boldsymbol{L}}$.

one can develop a form of the Laplacian method in which range-rate data may easily be introduced. The fundamental equations are

$$\left(\dot{\boldsymbol{L}} \cdot \dot{\boldsymbol{L}} - \frac{\mu}{r^3}\right) \varrho + \frac{\mu}{r^3} \boldsymbol{L} \cdot \boldsymbol{R} - \ddot{\varrho} + \boldsymbol{L} \cdot \ddot{\boldsymbol{R}} = O \tag{6}$$

and Eq. (2). All these quantities are defined at the second date, and $\ddot{\varrho}$ is obtained by numerical differentiation from a series of $\dot{\varrho}$'s. (See Ref. [*19*] for a derivation of the equations.) Eqs. (2) and (6) are solved simultaneously for ϱ and r by NEWTON's approximation. The method is particularly advantageous when there are rather large time intervals between obsevations, but one encounters difficulties due to an inaccurate determination of $\dot{\boldsymbol{L}}$ and especially $\ddot{\varrho}$ by numerical differentiation. Numerical differentiation of $\dot{\varrho}$, yielding $\ddot{\varrho}$, can be improved slightly by the following procedure (also developed in Ref. [*24*]):

$$\ddot{\varrho} = \frac{d}{d\tau} \dot{\boldsymbol{r}}(\cdot \boldsymbol{L}) + (\ddot{\boldsymbol{R}} \cdot L + \dot{\boldsymbol{R}} \cdot \dot{\boldsymbol{L}}) \quad \text{at time,} \quad t_2, \tag{6a}$$

where

$$\frac{d}{d\tau} (\dot{\boldsymbol{r}} \cdot \boldsymbol{L}) = [-\tau_{23}^2 (\dot{\boldsymbol{r}} \cdot \boldsymbol{L}) + (\tau_{23}^2 - \tau_{12}^2)\, (\dot{\boldsymbol{r}} \cdot \boldsymbol{L}) + \tau_{12}^2 (\dot{\boldsymbol{r}}_3 \cdot \boldsymbol{L}_3)] / \tau_{12}\, \tau_{23}\, \tau_{13},$$

with

$$\dot{\boldsymbol{r}}_i \cdot \boldsymbol{L}_i = \dot{\varrho}_i - \dot{\boldsymbol{R}}_i \cdot \boldsymbol{L}_i \quad \text{and} \quad \tau_{ij} \triangleq k(t_j - t_i), \qquad i, j = 1, 2, 3;$$

but the influence of a poorly defined $\dot{L}$ (i. e., $\dot{L}_2$) is still present.

Modified Lagrange-Gauss-Gibbs method [*24*]. There are two characteristics of deep space probe observations that, as we have seen, create difficulties in most preliminary orbit methods:

1. $\dot{\boldsymbol{L}}_2$ and $\ddot{\boldsymbol{L}}_2$ are poorly defined.
2. The unit vectors $\boldsymbol{L}_1$, $\boldsymbol{L}_2$, and $\boldsymbol{L}_3$ are all quite close together (nearly parallel).

In the modified LAGRANGE-GAUSS-GIBBS method these disadvantages are turned into advantages in that one factor tends to offset the other. The fundamental equations of of the method are

$$\varrho_2 = A + B\, \dot{\varrho}_1 + C\, \dot{\varrho}_2 + D\, \dot{\varrho}_3, \tag{7}$$

$$r_2\, \dot{r}_2 = \varrho_2\, \dot{\varrho}_2 - \boldsymbol{R}_2 \cdot (\dot{\varrho}_2\, \boldsymbol{L}_2 + \varrho_2\, \dot{\boldsymbol{L}}_2) + \varrho_2\, \boldsymbol{L}_2 \cdot \dot{\boldsymbol{R}}_2, \tag{8}$$

and Eq. (2). (The derivation of these equations is presented in Ref. [*24*].) The method simply involves an original guess of ϱ_2, a solution of Eq. (8) [to gain values for the coefficients in Eq. (7)], and then a *direct* (not an iterative) solution of Eq. (8). As can be seen by inspection of Eq. (8) and from the coefficients of Eq. (7), the influence of $\dot{\boldsymbol{L}}_2$ is greatly reduced. (See Appendix *A* for definition of the coefficients.) In Eq. (8), although

$\dot{\varrho}_2 \boldsymbol{L}_2$ and $\varrho_2 \dot{\boldsymbol{L}}_2$ are of about the same magnitude for the deep space probes (observed near the observer's zenith), $\boldsymbol{R}_2$ is nearly perpendicular to $\dot{\boldsymbol{L}}_2$, so that the adverse influence of an inaccurate value of $\dot{\boldsymbol{L}}_2$ is attenuated. Furthermore, only the product $(\boldsymbol{L}_1 - \boldsymbol{L}_3) \cdot \dot{\boldsymbol{L}}_2$ is found in the coefficients of Eq. (7), so that the influence of an inaccurate value of $\dot{\boldsymbol{L}}_2$ and the near equality of $\boldsymbol{L}_1$ and $\boldsymbol{L}_3$ cancel. Another advantage is that the method yields a solution for ϱ_2 directly and does not require the generation of a derivative for use in NEWTON's approximation. Disadvantages of the method include limitations on the time between observations. For space probes near the lunar distance, observations spaced further apart than about three days involve the accumulation of too much truncation error.

As in the case of the Laplacian method, a modified LEUSCHNER differential correction can be employed to improve agreement at the first and third dates [*24*].

General comments. Numerical comparisons of the foregoing methods for the Ranger III space probe may be found in Ref. [*24*]. It should be noted that the modified LAGRANGE-GAUSS-GIBBS method reduces to the modified Laplacian if the time intervals between observations approach zero.

Most preliminary orbit methods such as these discussed above and in Ref. [*25*] suffer from three classes of difficulties:

1. Observational error, both random and systematic.
2. Deviation from the usually assumed two-body, circular, or parabolic orbits, etc.
3. Ambiguities and indeterminacies inherent in the data or in the method.

The first difficulty can render a preliminary orbit method unusable if the errors are too large; very little can be done about this problem. With respect to 2, often an extension of the f and g series to include both in-plane perturbations such as drag and the aspherical Earth can be accomplished, as in Ref. [*1, 26*] and [*27*]. Furthermore, the f and g series can be extended, e. g.,

$$\dot{\boldsymbol{r}}_i = f_i\, \boldsymbol{r}_0 + g_i\, \dot{\boldsymbol{r}}_0 + h_i\, \boldsymbol{r}_0 \times \dot{\boldsymbol{r}}_0,$$

to take into account out-of-plane perturbations. Problems of ambiguities and indeterminacies, 3, are usually more subtle, and their solution involves considerable insight into the method being used. The vanishing of the D determinant in the Laplacian method for great circle orbital paths on the celestial sphere is one good example, and the four possible solutions for range-only orbit determination (see Ref. [*27*]) represents another interesting case. If one can settle for the solution of fewer than